



## A calculation of the load bearing capacity of glulam beams with BFRP reinforcement

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### ABSTRACT:

Strengthening timber elements using fiber composites increasingly being used in construction. Despite the increasing popularity of this solution, there are still no normative provisions. The article discusses the effectiveness of the most common method used to determine the load capacity of glued laminated timber beams reinforced with BFRP materials. Load capacity values, calculated analytically, were compared with the values determined in experimental tests. The method presented in the article may be helpful from an engineering perspective.

### KEYWORDS:

timber; fiber composites; BFRP; analytical calculations

## 1. Introduction

The calculation of timber elements reinforced with FRP materials is not regulated in Poland. An important issue in the design of elements, for which there are no normative provisions, is the possibility of using existing calculation algorithms to assess the load capacity. It is important that in the case of this type of calculation method, the complexity of both assumptions and mathematical calculation procedures should be small. Jasieńko [1] presented analytical methods for estimating the load-bearing capacity of timber elements reinforced with steel and attached wooden elements, while Nowak [2] reinforced with composites. Nowak also proposed a proprietary method for estimating the load capacity of timber beams reinforced with CFRP tapes. Currently, the literature usually presents methods using surrogate geometric and material characteristics [3] and the method based on the balance of forces in the cross section, such as presented in [4, 5].

The aim of the article is to assess the usefulness of the analytical method in estimating the load capacity of glued laminated timber, reinforced with BFRP bars.

## 2. Methodology

### 2.1. Experiment

For the analysis of analytically calculated values, the data presented in article [6] was used. The cited publication presents experimental studies of glued laminated timber beams reinforced with basalt-epoxy rods. GL24h wood (Table 1) and basalt epoxy BFRP bars (Table 2) were used to make the beams. The beams were subjected to a four-point bending test. The values of destructive forces are presented in Table 3.

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**Table 1**  
Characteristic values of strength and elasticity in N/mm<sup>2</sup> and density in kg/m<sup>3</sup> for homogeneous glued laminated wood [7]

| Properties            | Symbol          | Value  |
|-----------------------|-----------------|--------|
| Bending strength      | $f_{m,g,k}$     | 24     |
| Tensile strength      | $f_{t,0,g,k}$   | 19.2   |
|                       | $f_{t,90,g,k}$  | 0.5    |
| Compression strength  | $f_{c,0,g,k}$   | 24     |
|                       | $f_{c,90,g}$    | 2.5    |
| Shear strength        | $f_{v,g,k}$     | 3.5    |
| Modulus of elasticity | $E_{0,g,mean}$  | 11 500 |
|                       | $E_{0,g,05}$    | 9600   |
|                       | $E_{90,g,mean}$ | 300    |
|                       | $E_{90,g,05}$   | 250    |
| Modulus of rigidity   | $G_{g,mean}$    | 650    |
|                       | $G_{g,05}$      | 540    |
| Density               | $\rho_{g,k}$    | 405    |
|                       | $\rho_{g,mean}$ | 445    |

**Table 2**  
Characteristic values of strength and elasticity for BFRP basalt-epoxy rods [8-10]

| Properties  | Symbol         | Value [MPa] |
|---|----------------|-------------|
| Mean tensile strength for $\phi 7$ bars           | $\hat{f}_u$    | 1184.97     |
| Characteristic tensile strength for $\phi 7$ bars | $\hat{f}_{uk}$ | 1474.90     |
| Mean tensile strength for $\phi 9$ bars           | $\hat{f}_u$    | 817         |
| Characteristic tensile strength for $\phi 9$ bars | $\hat{f}_{uk}$ | 1038        |
| Modulus of elasticity $\phi 7$ for bars           | $E_{f,sr}$     | 52 800      |
| Modulus of elasticity for $\phi 9$ bars           |                | 56 300      |

**Table 3**  
Comparison of the values of destructive forces obtained in analytical calculations with the values obtained in experimental tests

| Series | Destructive force – analytical calculations | Destructive force – experimental tests | Difference | Mean difference |
|--------|---|--|------------|-----------------|
|        | (kN)  | (kN)                                   | (%)        | (%)             |
| A      | 53.32                                       | 55.25                                  | -3.61      | -17.06          |
|        | 53.32                                       | 56.26                                  | -5.51      |                 |
|        | 53.32                                       | 60.8                                   | -14.02     |                 |
| B      | 54.10                                       | 67.92                                  | -25.55     |                 |
|        | 54.10                                       | 64.78                                  | -19.75     |                 |
|        | 54.10                                       | 58.3                                   | -7.77      |                 |
| C      | 53.62                                       | 68.54                                  | -27.83     |                 |
|        | 53.62                                       | 65.42                                  | -22.02     |                 |
| D      | 53.79                                       | 65.9                                   | -22.51     |                 |
|        | 53.79                                       | 64.25                                  | -19.44     |                 |
|        | 53.79                                       | 64.39                                  | -19.70     |                 |

## 2.2. Analytical calculations

In order to estimate the load-bearing capacity of timber elements reinforced with fibrous composites, a popular method was used in the calculations that is often presented in the literature [4, 5]. The assumptions and calculation procedures of the said method match the analyzed structural elements.

The main assumptions of the adopted calculation model are shown in Figure 1. For the calculations, the elastic and ideal elastic nature of the wood in compression was adopted, whereas the ideal elasticity in tension (Fig. 2a) was used, which is related to the wood characteristics. For BFRP basalt-epoxy rods, the elastic model was adopted for both tensile and compression (Fig. 2b).

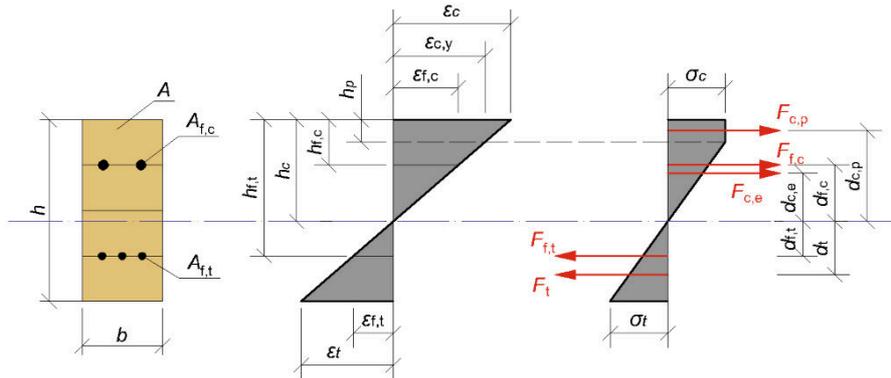


Fig. 1. The adopted assumptions of the analytical model in cross section

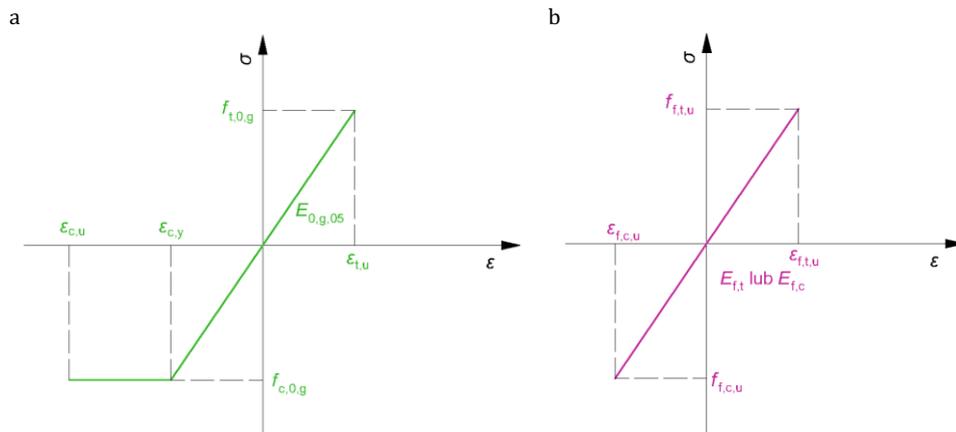


Fig. 2. Material models used for analytical calculations: a) glulam, b) BFRP bars

Values of wood strength were adopted based on standard data [7], which are presented in Table 1. Due to the small dimensions of the samples tested experimentally, the values of bending and tensile strength were increased by 10% ( $f_{m,g,ft,0,g}$ ), which is justified by the provisions of Eurocode 5 [11]. In addition, based on the recommendations presented in [5], a 10% increase in compressive strength ( $f_{c,0,g}$ ) was also assumed.

In the calculations, an increasing coefficient value ( $\alpha_m = 1.4$ ) of bending strength was used [5], due to the beneficial effect of the reinforcement on the form of destruction. Finally, the bending strength of timber reinforced with BFRP bars was calculated according to formula 1. The longitudinal modulus of wood  $E_{0,g,05}$  [7] was adopted, which allows the results closest to

experimental research to be obtained. The data adopted for basalt-epoxy rods are presented in Table 2, and average values were used for the calculations.

$$f_{m,g,r} = \alpha_m f_{m,g} = \alpha_m E_{0,g,05} \varepsilon_{m,u} \quad (1)$$

Due to the high strength of BFRP rods, two possible models of element destruction were assumed: by exceeding the limit of tensile strength of the wood fibres in the tension zone and by crushing timber fibres in the compression zone.

#### Load bearing capacity under tension

Based on the theoretical model (Fig. 1) and adopted material models (Fig. 2a, 2b) and the determination of the cross-sectional equilibrium (formula 2), it is possible to determine the position of the neutral axis ( $h_{c,t}$ ,  $h_{c,c}$ ). Calculation of the load capacity for both failure models ( $M_{t,f}$ ,  $M_{c,f}$ ) is possible after determining the position of the neutral axis.

$$F_t + F_{f,t} = F_{f,c} + F_{c,e} + F_{c,p} \quad (2)$$

$$F_{f,c} = A_{f,c} \sigma_{f,c} = A_{f,c} \varepsilon_{f,c} E_{f,c} \quad (3)$$

$$F_{c,p} = A_{c,p} f_{c,0,g} = b h_p E_{0,g,05} \varepsilon_c \quad (4)$$

$$F_{c,e} = \frac{A_{c,e} f_{c,0,g}}{2} = \frac{b(h_c - h_p) E_{0,g,05} \varepsilon_{c,y}}{2} \quad (5)$$

$$F_{f,t} = A_{f,t} \sigma_{f,t} = A_{f,t} \varepsilon_{f,t} E_{f,t} \quad (6)$$

$$F_t = \frac{A_t f_{t,0,g}}{2} = \frac{(h - h_c) b E_{0,g,05} \varepsilon_t}{2} \quad (7)$$

$$\varepsilon_c = \frac{h_c}{h - h_c} \varepsilon_t \quad (8)$$

$$\varepsilon_{c,y} = \frac{h_c - h_p}{h - h_c} \varepsilon_t \quad (9)$$

$$\varepsilon_{f,c} = \frac{h_c - h_{f,c}}{h - h_c} \varepsilon_t \quad (10)$$

$$\varepsilon_{f,t} = \frac{h_{f,t} - h_c}{h - h_c} \varepsilon_t \quad (11)$$

After substituting formulas 3 - 11 for equation 2 and transforming relative to the variable  $h_{c,f}$ , the quadratic equation can be obtained.

After determining the position of the neutral axis, it is possible to:

- determine the height of the plasticized zone,
- check the conditions for the deformation of reinforcing bars.

Knowing the position of the neutral axis and the height of the plasticized zone, it is possible to determine the cross-sectional force arm.

The tensile load failure model can be determined according to the formula:

$$M_{t/c,f} = F_{f,c} d_{f,c} + F_{c,p} d_{c,p} + F_{c,e} d_{c,e} + F_t d_t + F_{f,t} d_{f,t} \quad (12)$$

#### Load bearing capacity under compression

When compression failure is analyzed, it is assumed to exceed the limit value of plastic deformation for compression in timber. The value of plastic deformation has been assumed

equal to  $\varepsilon_{c,u} = 1.2\%$  and  $\varepsilon_{f,c} = 0$  on the basis of [5]. As in the previous model, the equilibrium equation 2 is used and the quadratic equation obtained.

The load capacity due to compression can be determined on the basis of formula (12).

The final value is the smaller of the two determined values for different destruction models:

(13)

### 3. Results

Based on the calculation procedures presented in the article, the values of destructive forces were determined for all series of reinforced beams. The comparison of the values of destructive forces determined theoretically and on the basis of experimental tests, together with the determination of the percentage differences in the results obtained is presented in Table 3. The correlation between the values from the experiment and the theoretical values is shown in Figure 3.

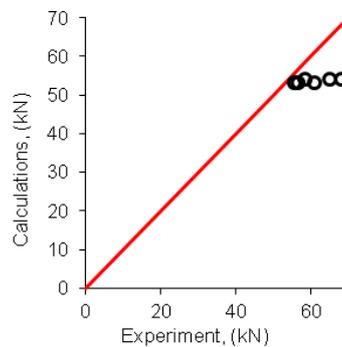


Fig. 3. Comparison of values of destructive forces obtained in analytical calculations and experimental tests

### 4. Discussion and conclusion

Based on the calculations carried out, it can be seen that:

- The average difference between the values of destructive forces obtained in experimental tests and theoretically determined values is -17.06%. The obtained results are comparable with the values presented in other publications [4, 5].
- Based on theoretical calculations, the load capacity of the element is underestimated in 94% of cases, which is beneficial in engineering applications from the point of view of structural safety.
- Using the presented method to estimate the load capacity of beams has application possibilities, but requires knowledge of the exact material characteristics of the designed elements.
- Assumptions were made in relation to the use of BFRP bars to strengthen the analyzed beams, for which there is no accurate material data. Therefore, the results obtained are not completely reliable.

### Literature

- [1] Jasięńko J., Połączenia klejowe w rehabilitacji i wzmacnianiu zginanych belek drewnianych, Oficyna Wydawnicza Politechniki Wrocławskiej, Wrocław 2002.
- [2] Nowak T., Analiza pracy statycznej zginanych belek drewnianych wzmacnianych przy użyciu CFRP, Rozprawa doktorska, Wrocław 2007.
- [3] Fossetti M., Minafò G., Papia M., Flexural behaviour of glulam timber beams reinforced with FRP cords, Construction and Building Materials 2015, 95, 54-64.

- [4] Borri A., Corradi M., Grazini A., A method for flexural reinforcement of old wood beams with CFRP materials, *Composites: Part B* 2005, 36, 143-153.
- [5] Yang H., Liu W., Lu W., Zhu S., Geng Q., Flexural behavior of FRP and steel reinforced glulam beams: Experimental and theoretical evaluation, *Construction and Building Materials* 2016, 106, 550-563.
- [6] Rajczyk M., Jończyk D., Badania belek z drewna klejonego warstwowo wzmocnionych prętami bazaltowo-epoksydowymi, *Zeszyty Naukowe Politechniki Częstochowskiej. Seria Budownictwo* 2018, 24, 298+304.
- [7] PN-EN 14080:2013 Konstrukcje drewniane -- Drewno klejone warstwowo i drewno lite klejone warstwowo – Wymagania.
- [8] Szymczak P., Olbryk P., Chołostiakow S., Kamińska M., Badanie betonowych belek zbrojonych prętami kompozytowymi, Wyniki badania belek zbrojonych prętami GFRP i BFRP. Sprawozdanie z badań.
- [9] Szymczak P., Olbryk P., Chołostiakow S., Kamińska M., Badanie przyczepności prętów kompozytowych GFRP i BFRP do betonu, Wyniki badania próbek zbrojonych prętami GFRP i BFRP. Sprawozdanie z badań.
- [10] Szymczak P., Olbryk P., Chołostiakow S., Kamińska M., Badanie prętów kompozytowych, Wyniki badania prętów kompozytowych GFRP i BFRP. Sprawozdanie z badań.
- [11] PN-EN 1995-1-1:2010 Eurokod 5: Projektowanie konstrukcji drewnianych – Część 1-1: Postanowienia ogólne i reguły dotyczące budynków.

## **Obliczanie nośności belek z drewna klejonego warstwowo zbrojonych BFRP**

### **STRESZCZENIE:**

Wzmacnianie elementów drewnianych z wykorzystaniem kompozytów włóknistych jest coraz szerzej stosowanym rozwiązaniem w budownictwie. Pomimo zwiększającej się popularności rozwiązania nadal brakuje przepisów normatywnych. W artykule oszacowano przydatność powszechnie znanej metody określania nośności wzmacnianych elementów drewnianych do obliczania belek z drewna klejonego warstwowo wzmacnianych materiałami BFRP. Porównano wartości nośności obliczone analitycznie z wartościami wyznaczonymi w badaniach eksperymentalnych. Metoda przedstawiona w artykule może być pomocna z inżynierskiego punktu widzenia.

### **SŁOWA KLUCZOWE:**

Drewno; kompozyty włókniste; BFRP; obliczenia analityczne