



The optimization of kinematic and geometric parameters in two-element grinding discs with a central rotational axis for the uniformity of concrete surface treatment

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ABSTRACT:

The last stage of grinding concrete surfaces is performed by floaters with solid grinding discs, where the working element consists of a solid wheel. The working element uses the full surface of the disc, ensuring maximum geometrical effectiveness. The disc rotates around its center and at the same time, it is in a steady progressive movement. The effectiveness chart S_g for the solid disc with common kinematic parameters has a local minimum at the inside and zero values on the edges of the machining area. Increasing the effectiveness of the machining at the edges is accomplished by overlapping areas of machining in subsequent cycles. Increasing the local minimum effectiveness can be achieved by applying a two-element disc consisting of concentric machining elements - ring and wheel. Optimal selection of the rotation speed and wheel radius gives better uniformity of two-element disc machining in comparison to the solid disc. The grinding uniformity of areas with widths smaller than the diameter of the disc can be achieved by machining them with the biggest effect uniformity area of action. The best effect can be achieved by using both methods at the same time.

KEYWORDS:

geometric effectiveness; floating; concrete; optimalization

1. Introduction

Disc trowels are used to float concrete surfaces. Their main advantages are high efficiency, simple construction and high reliability [1, 2]. To complete the final phase on concrete surfaces, solid discs with working elements in the shape of wheels are used. This process is called final mashing and is often combined with the surface refining process. The magnitude of the mashing disc impact determined on the machined point of the surface is called the geometric efficiency S_g . Due to the kinematic properties of the disk, which moves in a constant, straightline motion and at the same time rotates with a constant value, the geometric efficiency at the points of the surface treated by the central part of the disk is smaller than near the edges. This adversely affects the uniformity of machining. The method analyzed in the article that can reduce this effect is the separation of the working element in the form of a independent circle in the middle of the disk, which can move with a higher rotational velocity, increasing the value of the geometric efficiency in the central part of the disc and improving the uniformity of the entire system. This paper determines the optimal radius parameters of the selected circle in the middle of the disc and its rotational speed for providing the best uniformity of surface treatment.

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2. The geometric efficiency provided by solid disc motion on the treated surface

The parameter determining the amount of mashing required at a point on the treated surface is the geometric efficiency S_g defined as the surface contact line length of the mashing disk with the above mentioned point [3].

The end seizing disc has the shape of a circle. During troweling, it rotates in a constant motion around the axis located in the center of the circle and at the same time in a straight motion. The range of the disk impact is limited by the machined surface points located at a smaller distance from the center of the wheel than its radius.

The velocity vector resulting from the linear speed of the disc is constant at any point of the disc area impact.

The value of the linear speed vector resulting from the rotational disc speed is proportional to the rotational speed of the disc ω and the distance of the point from its center r . The direction of this vector is perpendicular to the radius of the outgoing point from center of rotation, and its direction depends on the rotation direction.

The total speed of the disc action V_w at any point is the sum of the vector of the straightline speed of the disc and the linear speed resulting from the rotational speed. The principle of creating a resultant velocity vector for a point in the area of disk impact is shown in Figure 1.

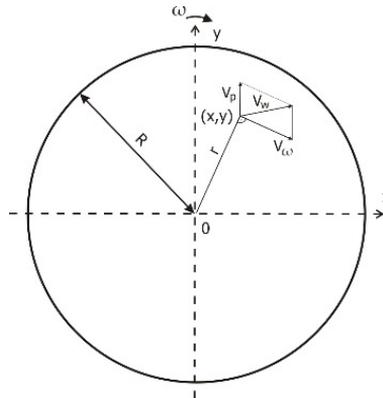


Fig. 1. Diagram of the folding of the resultant speed vector resulting from the progressive and rotational velocity of the disc for any point with coordinates relative to the center of the disc

The value of the resultant velocity vector V_w of the disc moving with a progressive movement velocity V_p and rotational speed ω at point P with coordinates x, y in the coordinate system with the origin in the center of the disc and the Y axis parallel to the progressive movement velocity vector V_p as shown in Figure 1 is given by the formula:

$$V_w(x, y) = \sqrt{V_p^2 + 2V_p x \omega + y^2 \omega^2 + x^2 \omega^2} \quad (1)$$

The geometric efficiency S_g of the wheel-shaped disk after its complete passage through the tested point $P(x, y)$ is the sum of the products of the accidental velocities of the impact V_w and the time of impact t :

$$S_g(x) = \int_{t_1}^{t_2} V_w(x, t) dt \quad (2)$$

After integration, we obtain a formula determining the geometric effectiveness of the full disc impact on the points of the machined surface depending on the cut off value in relation to the center of the disc:

$$S_g(x) = \frac{1}{V_p} \sqrt{(R^2 - x^2)(V_p^2 + 2V_p\omega x + \omega^2 R^2)} + \frac{(V_p + x\omega)^2}{2\omega V_p} \ln \left(\frac{\sqrt{V_p^2 + 2V_p\omega x + \omega^2 R^2} + \omega \sqrt{R^2 - x^2}}{\sqrt{V_p^2 + 2V_p\omega x + \omega^2 R^2} - \omega \sqrt{R^2 - x^2}} \right) \quad (3)$$

The formula applies to $x \notin (-R; R)$, $V_p > 0$ and $\omega \neq 0$.

For $x \notin (-R; R)$ the geometric efficiency value is $S_g = 0$.

For $V_p = 0$, the value of geometric efficiency is infinite, but this has no practical applications.

For rotational speed $\omega = 0$ the value of geometric efficiency can be calculated from the formula:

$$S_g(x) = 2\sqrt{R^2 - x^2} \quad (4)$$

Information about deriving the above formulas are presented in [4].

3. Geometric efficiency of the ring element

The geometrical effectiveness of the interaction of a ring element with an outer radius R_z and an inner radius R_w after a single pass of the disc can be calculated from the superposition principle by subtracting from the efficiency of the wheel with the radius R_z the efficiency of the wheel with the radius R_w as described:

$$S_g = S_{gR_z} - S_{gR_w}$$

where: S_{gR_z} - geometric efficiency calculated for a circle with a radius R_z , S_{gR_w} - geometric efficiency calculated for a circle with a radius R_w .

After substitutions and simplification, the formulas will take the following form for individual cases:

for $|x| \in (0; R_w)$

$$S_g(x) = \frac{1}{V_p} \left(\sqrt{(R_z^2 - x^2)(V_p^2 + 2V_p\omega x + \omega^2 R_z^2)} - \sqrt{(R_w^2 - x^2)(V_p^2 + 2V_p\omega x + \omega^2 R_w^2)} \right) + \frac{(V_p + x\omega)^2}{\omega V_p} \ln \left(\frac{\sqrt{V_p^2 + 2V_p\omega x + \omega^2 R_z^2} + \omega \sqrt{R_z^2 - x^2}}{\sqrt{V_p^2 + 2V_p\omega x + \omega^2 R_w^2} + \omega \sqrt{R_w^2 - x^2}} \right) \quad (5)$$

for $|x| \in (R_w; R_z)$ the formula takes the same form as for a circle

$$S_g = \frac{1}{V_p} \sqrt{(R_z^2 - x^2)(V_p^2 + 2V_p\omega x + \omega^2 R_z^2)} + \frac{(V_p + x\omega)^2}{2\omega V_p} \ln \left(\frac{\sqrt{V_p^2 + 2V_p\omega x + \omega^2 R_z^2} + \omega \sqrt{R_z^2 - x^2}}{\sqrt{V_p^2 + 2V_p\omega x + \omega^2 R_z^2} - \omega \sqrt{R_z^2 - x^2}} \right) \quad (6)$$

for $|x| > R_z$ the geometric efficiency value is $S_g = 0$.

4. Geometric efficiency of complex ring-circular systems

Systems composed of a centrally arranged ring and circular working elements with a common center of rotation, with surfaces arranged to not have common parts can be calculated from the principle of superposition by summing their interactions.

The value of geometric efficiency can be calculated from the formula:

$$S_g(x) = \sum_{i=1}^n S_{gi}(x) \tag{7}$$

5. The assessment of disc action uniformity

In order to determine the uniformity of the disc action, the standard deviation index ε was adopted, having a minimum value of zero for constant test values and greater than zero values with the spread of test values. The advantage is that it determines the relative uniformity in relation to the average value. The value of this indicator quantifies the uniformity of the disc's impact on the surface being processed.

The objective of the function increasing the uniformity of the disc's impact on the processed surface is to minimize the standard deviation index of geometric efficiency ε . For given values of geometrical efficiency specified in n points distributed evenly on the section in order to determine the value of the standard deviation index ε , the following formula was adopted:

$$\varepsilon = \frac{\sigma}{\bar{S}_g} = \sqrt{\frac{\frac{1}{2}(S_{g1} - \bar{S}_g)^2 + \sum_{i=2}^{n-1}(S_{gi} - \bar{S}_g)^2 + \frac{1}{2}(S_{gn} - \bar{S}_g)^2}{\bar{S}_g^2(n-1)}} \tag{8}$$

6. Distribution of geometric efficiency S_g for a solid disc

The graph of geometric efficiency after a single pass of the solid disc obtained according to formula (3) is shown in Figure 2. The value of efficiency at the edges of the impact area of the disc is zero, and near the edges there are two local maximums of the function with different values. The higher maximum value occurs on the side where the rotational speed vectors add up to the progressive movement velocity vector, and the smaller on the opposite side where the vectors are subtracted. There is a local minimum in the middle of the chart. In order to increase the uniformity of disc action for machined areas wider than the diameter of the disc, the path of the disc is determined so that the areas of impact partially overlap as shown. In the overlaid areas, the effectiveness of that effect adds up. The size of the overlays is set in a way that the uniformity of disc action in the repetitive part is as large as possible, i.e. the standard deviation index is the smallest. The graph of geometric efficiency S_g with the use of optimal overlays is shown in Figure 3, and the principle of summing graphs is presented in Figure 4.

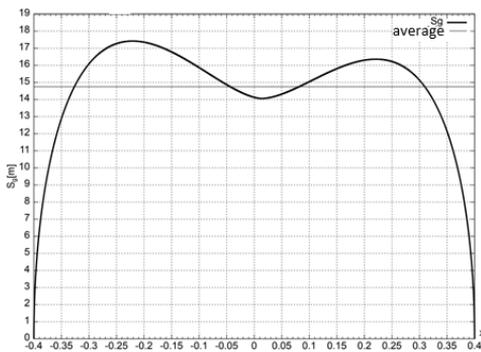


Fig. 2. Graph of geometric efficiency S_g for a single solid face transition $D = 0.8$ m, $V_p = 0.1$ m/s, $\omega = -8.8$ rad/s, $S_g = 14.7475$ m, $\varepsilon = 0.184$ m

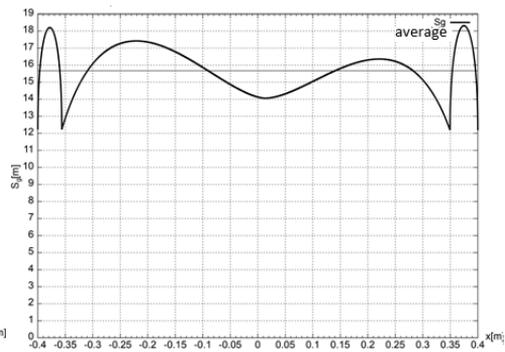


Fig. 3. Graph of geometric efficiency S_g for a solid disc with optimally applied machining zones $D = 0.6$ m, $V_p = 0.1$ m/s, $\omega = -8.8$ rad/s, $a = 0.043718$ m, $b = 0.504846$ m, $S_g = 15.6702$ m, $\varepsilon = 0.079$ m

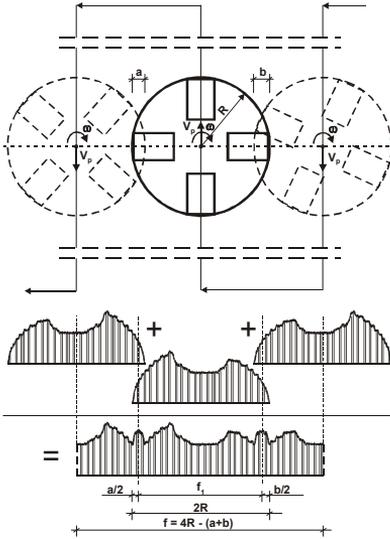


Fig. 4. The principle of using overlays to increase the uniformity of machining for rectilinear movement of the disc [5]

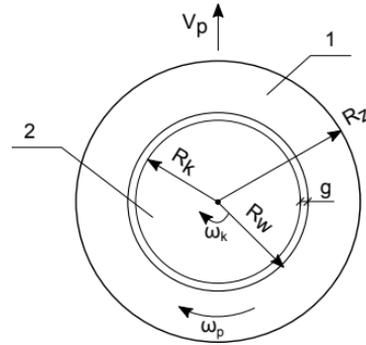


Fig. 5. Diagram of a two-element mashing disc with a central axis of rotation consisting of two working elements of the outer ring and the inner wheel that can move at independent angular speeds; 1 - ring, 2 - wheel

7. Geometry and kinematics of a two-element disc selected for the analysis

An analysis of the solid mashing disc action graph shape indicates the possibility of increasing the uniformity of machining by increasing the machining efficiency in the middle area of the graph.

This is possible when the homogeneous solid disc is replaced with a two-piece disc consisting of: a wheel located in the center of the disc's rotation and a ring surrounding the wheel so that they collectively fill the geometry of the solid disc, as shown in Figure 5. The wheel can rotate with a rotation velocity and direction different from the ring.

As an example, a solid trowel disc with the following parameters was adopted in the test:

progressive velocity	$V_p = 0.1 \text{ m/s}$
rotation velocity	$\omega_p = -8.8 \text{ rad/s}$ (-84 rpm/min, the disk rotates clockwise)
disc radius	$R_z = 400 \text{ mm}$.

The parameters of the ring from the two-element disc were adopted as constant values identical to the solid disc. The kinematic system from Figure 5 in terms of geometry and kinematics can be clearly defined when determining the other two parameters:

- the central radius of the circular work element R_k ,
- rotational velocity of the circular working element ω_k .

The inner radius of the ring R_w is dependent on the size of the wheel radius R_k by the following relationship $R_w = R_k + g$, where g is the distance between the circular and ring element.

Parameter values chosen for optimization:

- for technical reasons, it was assumed that the minimum thickness of the outer ring is 50 mm, which gives $R_w \leq 350 \text{ mm}$,
- the distance between the working elements taken for calculations $g = 1 \text{ mm}$,
- the minimum radius of the central, circular working element was $R_k = 10 \text{ mm}$,
- the maximum rotational velocity of the circular working element is selected so that the maximum linear speed of the circular element is not greater than the maximum progressive movement velocity of the ring element,
- the maximum size of the left and right side overlays is limited to the outer radius of the disc $R_w \leq 400 \text{ mm}$.

The geometric effectiveness of the S_g interaction of the discs on the machined surface was determined for 1601 points evenly distributed over the measuring section with a diameter of the disc equal to 800 mm set perpendicular to the direction of the progressive movement of the disc for rectilinear movement with optimal overlays.

The distribution of geometric efficiency for optimal disc parameters is shown in Figure 6. In this case, the value of the standard deviation index was $\varepsilon = 0.0696972$ m and was decreased to the comparative solid disc (which chart S_g is shown in Figure 3 for $\varepsilon = 0.079$ m). The chart in the middle shows an increase in geometric efficiency due to the operation of the circular element with increased rotational speed.

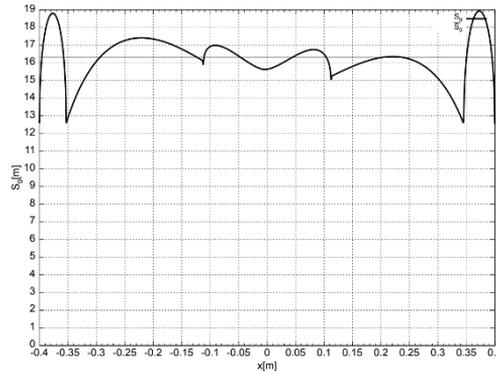


Fig. 6. Graph of geometric efficiency S_g for a two-element disc optimized for uniformity of machining with optimal overlays $D = 0.4$ m, $V_p = 0.1$ m/s, $\omega = -8.8$ rad/s, $a = 0.049$ m, $b = 0.052$ m, $S_g = 16.3081$ m, $\varepsilon = 0.0696972$ m

8. Optimization of disc parameters for machining areas with a width smaller than the diameter of the disc

The scheme of machining parameters selection for areas with a width smaller than the diameter of the disc is shown in Figure 7. The treatment corresponds to a single pass of the disc on the treated surface without overlays. The complete disc is set in relation to the machined area so that the continuous area of the plot with the width of the workpiece (the hatched part in the drawing) has the most even distribution. By modifying the parameters of a two-element disc, an optimized shape of the S_g plot is found for a given machining width. The ability to choose the position of the wheel relative to the surface to be machined is large when the machining area is small and decreases as the machining area increases. When the width of the machining area is equal to the diameter of the disc there is only one position that allows the machining of the entire area.

To optimize the system, a climbing algorithm was used for the analysed two-element disc and full disc in the range of the processed area width from 100 mm to the full diameter of the disc equal to 800 mm. Figure 8 shows the values of the standard deviation coefficient ε determining the equivalence of the impact distribution. In the whole area of the analysed machining widths, a two-element disc has greater uniformity than a solid disc. The values of the average efficiency for the optimized and solid disc are presented in Figure 9. The parameters of the radius of the inner wheel R_k and the angular velocity of the wheel ω_k are shown in Figures 10 and 11. Figure 12 shows the displacement of the center of the disc in relation to the center of the machined area width in the case of optimal uniformity of machining. In Figure 9, areas to the left and right of the graph with a value close to 11 m result from the implemented penalty function [6, 7] forcing efficacy over 11 m. Without the penalty function, the algorithm showed very small values of geometric efficiency, which would not allow practical application. The optimization was carried out for the maximum assumed velocity range in two directions (to the right and left). The results presented in Figure 11 show that for most of the I range the wheel velocity opposite to the ring velocity is more favorable.

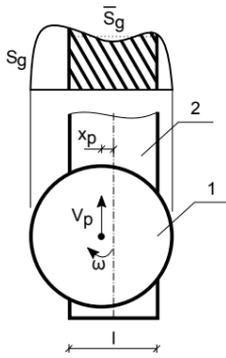


Fig. 7. Chosen principle scheme of the most even machining area with a width l of a seized linear prefabricated element with a width of the machining area smaller than the diameter of the grinding disk; 1 - scuffing disk, 2 - prefabricated

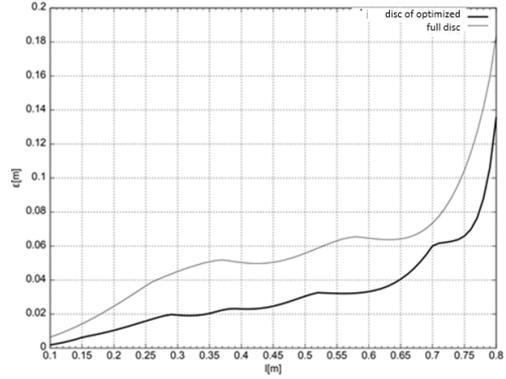


Fig. 8. Graph of standard deviation indicator ε in an optimally selected mashing range depending on the width of the interval l for the optimized disc and wheel; $D = 0.8$ m, $V_p = 0.1$ m/s, $\omega = -8.8$ rad/s

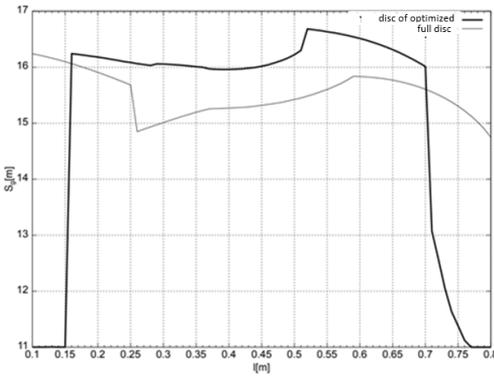


Fig. 9. Graph of the average geometric efficiency value S_g in the optimally selected mashing range depending on the width of the interval l for the optimized and solid disc; $D = 0.8$ m, $V_p = 0.1$ m/s, $\omega = -8.8$ rad/s

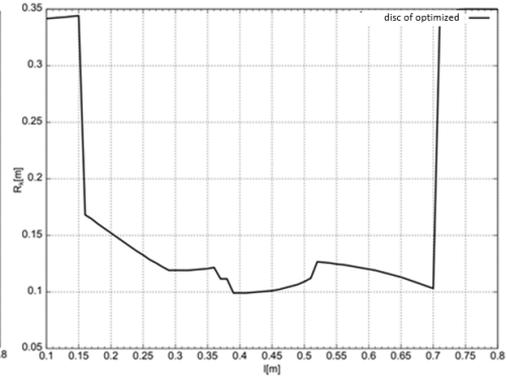


Fig. 10. Diagram of the radius of the inner wheel R_k in the disc optimized for the width of the mashing interval; $D = 0.8$ m, $V_p = 0.1$ m/s, $\omega_p = -8,8$ rad/s

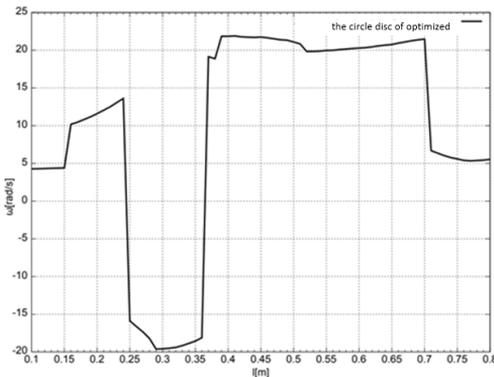


Fig. 11. The angular velocity of the inner wheel in the optimized wheel depending on the width of the machining interval; $D = 0.8$ m, $V_p = 0.1$ m/s, $\omega_p = -8.8$ rad/s

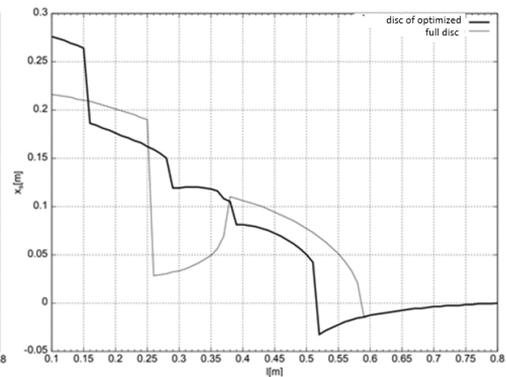


Fig. 12. Graph of the optimal x_p shift of the center of the mashing disc in relation to the symmetry axis of the machining area for the optimized and solid disc depending on the width of the machining area l ; $D = 0.8$ m, $V_p = 0.1$ m/s, $\omega_p = -8.8$ rad/s

9. Conclusions

The use of two-element mashing discs with a central axis of rotation allows the increase in the uniformity of surface treatment in comparison to a solid disc. For large-width machined surfaces that require troweling with overlays, the overall uniformity of machining is improved. The highest uniformity is obtained in the central part of the machining area, which is affected by the use of wheels with increased angular velocity in that area. Increasing the uniformity of machining in the overlay zone is small and results only from the increase in the average geometric efficiency for the entire machining area, which increases the size of the overlays. A much better effect is obtained when machining areas have their width smaller than the diameter of the mashing disc, e.g. in the case of surface treatment of prefabricated elements. Limiting the width of the area of action allows the selection of the continuous part in this area with the greatest uniformity. Optimal selection of geometrical and kinematic parameters significantly increases the uniformity of machining in relation to the solid disc in each width of the machining area. The use of an optimized two-element disc according to the analysed scheme for surface treatment with a width smaller than the diameter of the disc results in the increase of the quality of the machined surface.

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Optymalizacja parametrów kinematycznych i geometrycznych tarcz zacierających dwuelementowych o centralnej osi obrotu ze względu na równomierność obróbki powierzchni betonowych

STRESZCZENIE:

Ostatni etap zacierania powierzchni betonowych wykonuje się zacieraczkami z tarczami pełnymi, gdzie elementem roboczym jest koło. Element roboczy o takim kształcie wykorzystuje pełną powierzchnię tarczy, zapewniając jej maksymalną skuteczność geometryczną. Tarcza porusza się ruchem obrotowym wokół jej środka i jednocześnie ruchem jednostajnym postępowym. Wykres skuteczności geometrycznej S_g dla tarczy pełnej dla typowych parametrów kinematycznych posiada minimum lokalne w środku i wartości zerowe na krawędzi obszaru obróbki. Zwiększenie skuteczności obróbki na krawędziach uzyskuje się przez zastosowanie nakładających się stref obróbki w kolejnych cyklach. Likwidację lokalnego minimum skuteczności można uzyskać przez zastosowanie dwuelementowej tarczy składającej się z współśrodkowych elementów roboczych pierścienia i koła. Optymalny dobór prędkości obrotowej i promienia koła daje większą równomierność obróbki tarczy dwuelementowej w porównaniu z tarczą pełną. Równomierność zacierania obszarów o szerokościach mniejszych od średnicy tarczy można zwiększyć przez obróbkę ich obszarem tarczy o największej równomierności oddziaływania. Najlepsze wyniki uzyskuje się, stosując obydwie ww. metody jednocześnie.

SŁOWA KLUCZOWE:

skuteczność geometryczna; zacieranie; beton; optymalizacja