

Vitalii Lysenko, Dmytro Komarchuk

MATHEMATICAL MODELING OF THE RAPE GRAIN THERMAL PROCESSING IN A DENSE LAYER USING INDUCTION HEATING

Recently there was a clear trend of pre-heating oil raw material before it is fed to the compression [1-3]. Providing the recommended temperature increases the yield of oil and extends the life of the processing equipment [4-6]. The advantage of this technology is the uniformity of heating the disperse material in the layer under continuous stirring. But the technical realization and hardware implementation of the conductive heating method on the disc heating surface reduces the energy efficiency of the process because of the heat loss of natural convection and radiation into the environment and the vessel wall. In addition, all drawbacks of element electrical heating of working surfaces are presented.

To eliminate these shortcomings and improve the energy efficiency of the process, we propose a new method of heat processing of disperse material with the induction method for supplying energy to the heating surfaces [7]. The method provides that in a cylindrical supply channel of disperse material to the press-extruder there are mounted rotatably ferromagnetic core elements, and on the outer surface of the grain-driving channel an inductor is set [8-11]. The electric power supplied to the inductor and released in the heating elements arranged in a layer of grain is almost completely transferred to the grain material, and the convective and radiative components are also absorbed by the grain.

The proposed plant for the heat processing of rape grain [7] can provide the necessary temperature, but there is a need to create a mathematical model for the study of thermal processes occurring in it, and determine the influence of process parameters on the final temperature of rape grain.

A quantitative description of the thermal component dynamics of disperse material processing (rape grain) in a flow-type apparatus with electromagnetic heating elements can be formed on the basis of the mechanism of heat transfer [12-14]: the heat in the heating element (rod in the electromagnetic field) is transmitted by complex heat transfer (convection through inside-layer air, heat transfer by direct contact of rape grain and rods, radiation due to multiple reflection and absorption of radiant energy by disorderly located parts) to disperse material under the non-stationary conditions, given its longitudinal mixing relative to the surface of

heaters; the received heat of the material is spent on increasing its temperature and evaporation (residual moisture); during stirring of the material some part of heat is transferred to the frame walls, is consumed for heating and partially transmitted through the inductor winding to the environment; wherein the inductor winding increases its temperature. Transfer of moisture released from the material surface in interaggregate airspace material to material layer takes place by mass transfer diffusion and further through flow gaps between the particles to the environment.

The heat balance in the dynamics for an infinitesimal element of the heater filled with disperse material moving along the heaters (while stirring) for each of the following dynamic elements: the rod, the grain material, the air (the space between the grains), the body of the unit, inductor winding, we can write (for height dx of the elementary volume in the direction of material movement):

$$dQ_1 = dQ_{n.c.} + dQ_{c.3.} + dQ_{c.n.} \quad (1)$$

$$dQ_{c.3.} = dQ_{n.3.} + dQ_{6.6.} + dQ_{3.k.} + dQ_{3.n.} \quad (2)$$

$$dQ_{c.n.} + dQ_{3.n.} = dQ_{n.n.} + dQ_{n.k.} \quad (3)$$

$$dQ_{3.k.} = dQ_{n.k.} + dQ_{n.i.} + dQ_{i.o.} \quad (4)$$

where: Q_1 - the heat releasing in the heating elements; $Q_{n.c.}$ - the heat consumed to increase the temperature of the heating elements; $Q_{c.3.}$ - the heat transferred to material from the heating elements; $Q_{c.n.}$ - the heat received by the air between the grains; $Q_{n.3.}$ - the heat received by material to increase its temperature; $Q_{6.6.}$ - the heat that is spent on evaporation; $Q_{3.n.}$, $Q_{3.k.}$ - amount of the heat transmitted from the grain to the air and the unit frame; $Q_{n.n.}$ - the heat consumed for heating the air in the space between the grain; $Q_{n.k.}$ - the heat which is transferred from the air to the unit wall; $Q_{3.k.}$, Q_n - the heat which is transmitted to the unit frame; $Q_{n.k.}$, $Q_{n.i.}$ - amount of the heat for heating the unit frame and the inductor; $Q_{i.o.}$ - the heat consumed in the environment.

Thus, the modeled object is represented by four dynamic elements - thermal capacitances.

Before opening the components of heat balance equations we take the usual one-dimensional case simplifying assumptions:

- temperature gradient over the cross-section (radius) of the material flow, rods, the frame and inductor walls has a very small (minor) value and it can be neglected, i.e. contribution of the thermal conductivity to the calculations will be negligible compared to the effective heat transfer, thereby we accept a uniform temperature field along the radius of the chamber, which reduces the dynamic problem to one-dimensional case;
- heat transfer by conduction in the direction of material flow is negligible and can be neglected;

- thermal parameters of all elements of the object do not depend on the temperature and time;
- heat generation in the first approximation is assumed to be constant;
- given the intense heat supply and mixing of the material by heated rods, the temperature of the individual grains is assumed to be the same on the surface and in the center;
- the effects of radiation, conduction in the space between the rods and the convective heat transfer are taken into account due to the heat transfer coefficient:

$$\alpha_{efp} = \alpha_{\kappa} + \alpha_{\lambda} + \alpha_{\epsilon} \quad (5)$$

where: α_{κ} - the coefficient of heat transfer by convection; α_{λ} - the coefficient of heat transfer by thermal conductivity; α_{ϵ} - the coefficient of heat transfer by radiation.

The individual components of the equivalent heat transfer coefficient are defined as:

$$\alpha_{efp} = \frac{1}{\frac{\delta_1}{\lambda_{\kappa}} + \frac{\delta}{\lambda_p} + \frac{\delta_n}{\lambda}} \quad (6)$$

where: λ_{κ} - the reduced layer thermal conductivity at convective air flow through the disperse material layer; λ_p - effective radiative thermal conductivity; λ - thermal conductivity coefficient of the contact layer; δ_1 - half of the distance between the rods; δ - thickness of the air layer.

Determination of the coefficients can be estimated due to well-known expressions [15, 16]:

$$\lambda_{\kappa} = \frac{q\delta}{t_2 - t_1} \quad (7)$$

$$\lambda_p = \frac{3,46Xt_{cp}d(3m\epsilon_n + (1-m)\epsilon_m)}{1 + (1-m)(1-\epsilon_m)} \quad (8)$$

where: q - the intensity of the heat flow from the rod; $(t_2 - t_1)$ - the temperature difference between the rod and the material at a distance $\frac{\delta}{2}$; d - diameter of grains; m - the porosity of the layer; ϵ_n, ϵ_m - blackness degree of steam and material; t_{cp} - average temperature, $t_{cp} = (t_2 - t_1)0.5$.

For the selected element with height dx , revealing the components of the heat balance (1-4) explicitly, we describe the dynamics of heat transfer due to the system of differential equations.

For the rods:

$$c_c S_c \rho_c dx d\Theta_c = q_x dx d\tau - \frac{\alpha_{e\phi} f_e}{H} (\Theta_c - t_3) dx d\tau - \frac{\alpha_u f_e}{H} (\Theta_c - t_v) dx d\tau \quad (9)$$

for the material:

$$c_3 S_3 \rho_3 dx dt_3 = \frac{\alpha_{e\phi} f_c}{H} (\Theta_c - t_3) dx d\tau - r_0 c_3 S_3 \rho_{30} du dx - \frac{\alpha_\kappa f_3}{H} (t_3 - t_6) dx d\tau - \frac{\alpha_{e\phi} f_\kappa}{H} (t_3 - \Theta_\kappa) dx d\tau \quad (10)$$

for the air between the grains:

$$c_p S_3 \varepsilon_3 \rho_6 dx dt_6 = \frac{\alpha_\kappa f_c}{H} (t_3 - t_6) dx d\tau + \frac{\alpha_\kappa f_c}{H} (\Theta_c - t_6) dx d\tau + \frac{\alpha_\kappa f_\kappa}{H} (t_6 - \Theta_\kappa) dx d\tau \quad (11)$$

for the frame with the inductor:

$$c_\kappa S_\kappa \rho_\kappa dx d\Theta_\kappa = \frac{\alpha_{e\phi} f_\kappa}{H} (t_3 - \Theta_\kappa) + \frac{\alpha_\kappa f_\kappa}{H} (t_6 - \Theta_\kappa) dx d\tau - \frac{\alpha_3 f_{3\kappa}}{H} (\Theta_\kappa - t_0) dx d\tau \quad (12)$$

for the moisture in the grain:

$$S_3 \rho_{30} du dx = k f_3 (P_u(t_3) - P_6) \frac{1}{H} dx d\tau \quad (13)$$

where: c_c, c_3, c_p, c_κ - the specific heat capacity of the rods, the material, the air, the frame material and the inductor respectively; $\rho_c, \rho_3, \rho_6, \rho_\kappa, \rho_{30}$ - the density of the rod material, the grain, the air, the frame material, completely dry matter respectively; q_x - the specific power of heaters, $q_x = \frac{P}{H}$; S_c, S_3, S_κ - the sectional area of the rods, the material, the frame with the inductor; ε - the porosity of the grain layer; $f_c, f_3, f_\kappa, f_{3\kappa}$ - the surface area of the rods, the material, the internal frame and the external inductor; $\alpha_{e\phi}, \alpha_\kappa$ - the coefficient of effective and convective heat emission; H - the height of the heater; x - the coordinate in the direction of the material movement; τ - time; $\Theta_c, t_3, t_6, \Theta_\kappa, t_0$ - temperature of the rods, the material, the air, the frame, the environment respectively; u - the moisture content of the material; r_0 - the specific heat of vaporization; k - the coefficient of mass transfer; P_u, P_6 - the partial pressure of saturated air at the material surface temperature and in the air.

Using the technique of recording the differential equations in partial derivatives we make the appropriate conversion of equations (9-13).

We expand the total differential of the rod temperature in equation (9):

$$d\Theta_c = \frac{\partial\Theta_c}{\partial\tau} d\tau + \frac{\partial\Theta_c}{\partial x} dx \quad (14)$$

Given that $\frac{\partial x}{\partial\tau} = v_c = 0$, $S_c \rho_3 H = n$,

where: v_c - velocity of travel; $\frac{d\Theta_c}{d\tau} = \frac{\partial\Theta_c}{\partial\tau}$, rewrite equation (9):

$$m_c c_c \frac{\partial\Theta_c}{\partial\tau} = q_x - \alpha_{ep} f_c(\Theta_c - t_3) - \alpha_{\kappa} f_3(\Theta_c - t_v) \quad (15)$$

We expand the total differential of the grain temperature: $dt_3 = \frac{\partial t_3}{\partial\tau} d\tau + \frac{\partial t_3}{\partial x} dx$, divide the equation by dx : $\frac{dt_3}{dx} = \frac{dt_3}{d\tau} \frac{d\tau}{dx} + \frac{dt_3}{dx}$; denote $\frac{dx}{d\tau} = v_3$ - speed of grain movement.

Then the temperature gradient is expressed by the equation $\frac{dt_3}{dx} = v_3 \frac{\partial t_3}{\partial\tau} + \frac{\partial t_3}{\partial x}$.

Substituting this value in equation (13) and using the obvious relations: productivity $G_3 = v_3 S_3 \rho_3$, the layer volume $V_3 = SH$, weight of grain $m_3 = V_3 \rho_3$ rewrite (10) as follows:

$$c_3 m_3 \frac{\partial t_3}{\partial\tau} + c_3 G_3 H \frac{\partial t_3}{\partial x} = \alpha_{ep} f_c(\Theta_c - t_3) - \alpha_{\kappa} f_3(t_3 - t_6) - \alpha_{ep} f_{\kappa}(t_3 - \Theta_{\kappa}) - r_0 m_{30} \frac{\partial u}{\partial\tau} - c_3 G_3 H \frac{\partial u}{\partial\tau} \quad (16)$$

Similarly transform the equations (11-13):

$$m_6 c_p \frac{\partial t_6}{\partial\tau} = \alpha_{\kappa} f_c(t_3 - t_6) + \alpha_{\kappa} f_c(\Theta_c - t_6) + \alpha_{\kappa} f_{\kappa}(t_6 - \Theta_{\kappa}) \quad (17)$$

$$(m_{\kappa} c_{\kappa} + m_i c_i) \frac{\partial\Theta_{\kappa}}{\partial\tau} = \alpha_{ep} f_{\kappa}(t_3 - \Theta_{\kappa}) + \alpha_{\kappa} f_{\kappa}(t_6 - \Theta_{\kappa}) - \alpha_{\kappa} f_{\kappa 3}(\Theta_{\kappa} - t_0) \quad (18)$$

$$m_{03} \frac{\partial u}{\partial\tau} + G_{30} H \frac{\partial u}{\partial x} = k f_3(P_H(t_3) - P_V) \quad (19)$$

Thus, the mathematical model of thermal processes in the heater in the form of five differential equations in partial derivatives in accordance with the quantity of heat capacities of elementary components is determined.

To simplify the solution of the problem of continuous heating of grain in the process of movement, remaining however in almost acceptable range, it is held lowering the order of the differential equations.

Mathematical justification of the physics equations of heat transfer are such additional conditions:

- 1) heat conductivity effects and the heat radiation in the radial (relative to axis of the rod) can be considered by equivalent heat emission coefficient determined experimentally. Stacked heat in a small volume of air can be clamped neglected, i.e. take its heat capacity to the heat capacity of the material layer.
- 2) since the inductor winding structurally formed integrally with the frame, and the inductor itself is heat emission element, the increase in heat of the construction through heat emission from the moving grain layer can be neglected.
- 3) to close the system of equations when you exclude the derivatives, remain algebraic equations of the relationship between the parameters reflecting the course of the process, equation (19) actually describes the increase in the rate of drying at temperature increase of the grain that at low moisture content of 6-8% does not correspond to the experimental data (since the rate of drying reduces with time), the adoption of the assumption of proportionality rate of drying the heating rate will be practically justified, the latter assumption makes it possible to use in the calculation Renbinder criterion $Rb = \frac{cd\Theta}{rdu}$ and make the corresponding substitution $du = \frac{c}{rRb} dt_3$ in the equation (16).

Thus, taking into account the imposed additional conditions simplifying the mathematical description of the heat transfer processes, the dynamics of thermal processes of flow grain heater we present a mathematical model in the form of two differential equations with partial derivatives and constraint equations (i.e. as 2-capacitance object):

$$\begin{cases} m_c c_c \frac{\partial \Theta_c}{\partial \tau} = P_n \eta - \alpha_{eph} f_c(\Theta_c - t_3) - \alpha_\kappa f_3(\Theta_c - t_6) \\ c_3' m_3' \left(1 + \frac{1}{Rb}\right) \frac{\partial t_3}{\partial \tau} + c_3 G_3 H \left(1 + \frac{1}{Rb}\right) \frac{\partial t_3}{\partial x} = \alpha_{eph} f_c(\Theta_c - t_3) - \alpha_\kappa f_3(\Theta_c - t_6) - \alpha_{eph}' f_\kappa(t_3 - \Theta_\kappa) \end{cases} \quad (20)$$

$$\begin{cases} \alpha_\kappa f_3(t_3 - t_6) + \alpha_\kappa f_c(\Theta_c - t_6) + \alpha_\kappa' f_\kappa(t_6 - \Theta_\kappa) = 0 \\ \alpha_{eph}' f_\kappa(t_3 - \Theta_\kappa) + \alpha_\kappa f_\kappa(t_6 - \Theta_\kappa) - \alpha_\kappa f_\kappa(\Theta_\kappa - t_0) = 0 \end{cases} \quad (21)$$

The system of equations (20) contains four unknowns $\Theta_c, \Theta_3, t_6, \Theta_\kappa$. The values of air temperature and the frame are defined from the system of equations (21):

$$t_6 = e_1 t_3 + e_2 \Theta_c + e_3 t_0 \quad (22)$$

$$\Theta_\kappa = e_4 t_3 + e_5 t_0 + e_6 \Theta_c \quad (23)$$

where:

$$e_1 = \frac{\frac{\alpha_\kappa f_3}{\alpha_\kappa f_3 + \alpha_\kappa f_c - \alpha'_\kappa f_\kappa} - \frac{\alpha'_\kappa f_\kappa}{\alpha_\kappa f_3 + \alpha_\kappa f_c - \alpha'_\kappa f_\kappa} \frac{\alpha_{eph} f_\kappa}{\alpha_{eph} f_\kappa + \alpha_\kappa f_\kappa + \alpha_\kappa f_{\kappa 0}}}{L + \frac{\alpha'_\kappa f_\kappa}{\alpha_\kappa f_3 + \alpha_\kappa f_c - \alpha'_\kappa f_\kappa} \frac{\alpha_\kappa f_\kappa}{\alpha_{eph} f_\kappa + \alpha_\kappa f_\kappa + \alpha_\kappa f_{\kappa 0}}}$$

$$e_2 = \frac{\frac{\alpha_\kappa f_3 + \alpha_\kappa f_c - \alpha'_\kappa f_\kappa}{\alpha'_\kappa f_\kappa} \frac{\alpha_\kappa f_\kappa}{\alpha_\kappa f_3 + \alpha_\kappa f_c - \alpha'_\kappa f_\kappa}}{1 + \frac{\alpha'_\kappa f_\kappa}{\alpha_\kappa f_3 + \alpha_\kappa f_c - \alpha'_\kappa f_\kappa} \frac{\alpha_\kappa f_\kappa}{\alpha_{eph} f_\kappa + \alpha_\kappa f_\kappa + \alpha_\kappa f_{\kappa 0}}}$$

$$e_3 = \frac{\frac{\alpha'_\kappa f_\kappa \alpha_\kappa f_{\kappa 0}}{\left(\alpha_\kappa f_3 + \alpha_\kappa f_c - \alpha'_\kappa f_\kappa\right) \left(\alpha_{eph} f_\kappa + \alpha_\kappa f_\kappa + \alpha_\kappa f_{\kappa 0}\right)}}{1 + \frac{\alpha'_\kappa f_\kappa \alpha_\kappa f_\kappa}{\left(\alpha_\kappa f_3 + \alpha_\kappa f_c - \alpha'_\kappa f_\kappa\right) \left(\alpha_{eph} f_\kappa + \alpha_\kappa f_\kappa + \alpha_\kappa f_{\kappa 0}\right)}}$$

$$e_4 = \frac{\frac{\alpha_{eph} f_\kappa}{\alpha_{eph} f_\kappa + \alpha_\kappa f_\kappa + \alpha_\kappa f_{\kappa 0}} + \frac{\alpha_\kappa f_\kappa e_1}{\alpha_{eph} f_\kappa + \alpha_\kappa f_\kappa + \alpha_\kappa f_{\kappa 0}}}{\alpha_{eph} f_\kappa + \alpha_\kappa f_\kappa + \alpha_\kappa f_{\kappa 0}}$$

$$e_5 = \frac{\frac{\alpha_\kappa f_{\kappa 0}}{\alpha_{eph} f_\kappa + \alpha_\kappa f_\kappa + \alpha_\kappa f_{\kappa 0}} + \frac{\alpha_\kappa f_\kappa e_2}{\alpha_{eph} f_\kappa + \alpha_\kappa f_\kappa + \alpha_\kappa f_{\kappa 0}}}{\alpha_{eph} f_\kappa + \alpha_\kappa f_\kappa + \alpha_\kappa f_{\kappa 0}}$$

$$e_6 = \frac{\alpha_\kappa f_\kappa e_2}{\alpha_{eph} f_\kappa + \alpha_\kappa f_\kappa + \alpha_\kappa f_{\kappa 0}}$$

Substituting the value t_v and Θ_k from (20) after the transformations we obtain:

$$\left\{ \begin{array}{l} T_1 \frac{\partial \Theta_c}{\partial \tau} = P\eta - \frac{a_1}{b_1} \Theta_c + \frac{c_1}{b_1} + t_3 \end{array} \right. \quad (24)$$

$$\left\{ \begin{array}{l} T_2 \frac{\partial \Theta_3}{\partial \tau} + T_x \frac{\partial t_3}{\partial x} = -\frac{b_2}{a_2} t_3 + \frac{c_2}{a_2} + \Theta_c \end{array} \right. \quad (25)$$

In equations (24) and (25) is designated:

$$a_1 = \alpha_{eph} + \alpha_\kappa f_3 - \alpha_\kappa f_3 e_2$$

$$b_1 = \alpha_{eph} f_c + \alpha_\kappa f_3 e_1$$

$$c_1 = \alpha_\kappa f_3 e_3 t_0$$

$$a_2 = \alpha_{eph} f_c + \alpha_\kappa f_3 e_2 + \alpha_{eph} f_\kappa e_6$$

$$b_2 = \alpha_{eph} f_c + \alpha_\kappa f_3 + \alpha_{eph} f_\kappa + \alpha_\kappa f_3 e_1 + \alpha_{eph} f_\kappa e_4$$

$$c_2 = \alpha_\kappa f_3 e_3 + \alpha_{eph} f_\kappa e_5$$

$$T_1 = \frac{m_c c_c}{b_1}$$

$$T_2 = \frac{(c_3 m_3 + c_6 m_6)(Rd + 1)}{Rba_2}$$

$$T_x = \frac{c_3 G_3 H(Rd + 1)}{Rba_2}$$

Since the system of equations (24-25) has no strictly analytical solution, we determine an approximate solution.

Consider the steady state operation of the heater, i.e. assume $\partial\Theta/d\tau=0$, $\partial t_3/d\tau=0$ and obtain static characteristic of the heater - the distribution of the grain temperature with the height of heating chamber in the form of an ordinary differential equation.

By equation (24) the amount of the heater temperature Θ_c and substituting this value in (25) we obtain the following equation:

$$\Theta_c = \frac{b_1}{a_1} P\eta + \frac{b_1}{a_1} \frac{c_1}{b_1} + \frac{b_1}{a_1} t_3 \quad (26)$$

$$T_x \frac{dt_3}{dx} = \frac{-b_2}{a_2} t_3 + \frac{c_2}{a_2} + \frac{b_1}{a_1} P\eta + \frac{c_1}{a_1} + \frac{b_1}{a_1} t_3 \quad (27)$$

$$T_x \frac{dt_3}{dx} = -\left(\frac{b_2}{a_2} - \frac{b_1}{a_1}\right) t_3 + \frac{c_2}{a_2} + \frac{c_1}{a_1} + \frac{b_1}{a_1} P\eta \quad (28)$$

Introduce the designation:

$$A = \frac{c_2}{a_2} + \frac{c_1}{a_1} + \frac{b_1}{a_1} P\eta \quad (29)$$

$$B = \frac{b_2}{a_2} - \frac{b_1}{a_1} \quad (30)$$

The solution of equation (28) taking into account the notation adopted for the boundary conditions: $x=0$, $t_3=t_{31}$ (where t_{31} - the value of the grain temperature at the inlet to the heater), we obtain in the form:

$$t_3(x) = \frac{A}{B} - \left(\frac{A}{B} - t_{31}\right) e^{-\frac{B}{T_x}x} \quad (31)$$

Equation (31) determines the grain temperature distribution in the direction of its movement along the heating rods in the steady state.

Having determined the derivative from $t_3(x)$, we obtain the value of the temperature gradient of grain layer:

$$\frac{dt_3}{dx} = \frac{B}{T_x} \left(\frac{A}{B} - t_{31} \right) e^{-\frac{B}{T_x}x} \quad (32)$$

We substitute the value $\frac{dt_3}{dx}$ into the equation (25), obtain the equation with conventional derivatives:

$$\left\{ \begin{array}{l} T_1 \frac{d\Theta_c}{d\tau} - c_3 + \frac{a_1}{b_1} \Theta_c = t_3 \\ T_2 \frac{dt_3}{d\tau} + \frac{b_2}{a_2} t_3 + c_4 = \Theta_c \end{array} \right. \quad (33)$$

$$\left\{ \begin{array}{l} T_1 \frac{d\Theta_c}{d\tau} - c_3 + \frac{a_1}{b_1} \Theta_c = t_3 \\ T_2 \frac{dt_3}{d\tau} + \frac{b_2}{a_2} t_3 + c_4 = \Theta_c \end{array} \right. \quad (34)$$

where:

$$c_3 = P\eta + \frac{c_1}{b_1};$$

$$c_4 = B \left(\frac{A}{B} - t_{31} \right) e^{-\frac{B}{T_x}x} - \frac{c_2}{a_2};$$

We differentiate (34) with respect to time:

$$\frac{d\Theta_c}{d\tau} = T_2 \frac{d^2t_3}{d\tau^2} + \frac{b_2}{a_2} \frac{dt_3}{d\tau} \quad (35)$$

We substitute the value Θ_c and $\frac{d\Theta_c}{d\tau}$ in equation (33), after transformations we have the equation of the grain temperature changes with time:

$$T_1 T_2 \frac{d^2t_3}{d\tau^2} + \left(T_1 \frac{b_2}{a_2} + \frac{a_1}{b_1} T_2 \right) \frac{dt_3}{d\tau} + \left(\frac{a_1}{b_1} \frac{b_2}{a_2} - 1 \right) t_3 = c_3 + \frac{c_2}{a_2} \frac{a_1}{b_1} - \frac{a_1}{b_1} f(x) \quad (36)$$

We rewrite (36) in the form:

$$A \frac{d^2t_3}{d\tau^2} + B \frac{dt_3}{d\tau} + C t_3 = D_1(x) \quad (37)$$

where:

$$A = T_1 T_2;$$

$$B = T_1 \frac{b_2}{a_2} + \frac{a_1}{b_1} T_2;$$

$$C = \frac{a_1 b_2}{b_1 a_2} - 1;$$

$$D_1(x) = \frac{c_2 a_1}{a_2 b_1} - \frac{a_1}{b_1} f(x).$$

Solution of the inhomogeneous differential equation of the second order we get as the sum of solutions of the homogeneous and particular:

$$t(\tau, x) = c_1 e^{r_1 \tau} + c_2 e^{r_2 \tau} + \frac{D_1(x)}{C} \tag{38}$$

where:

c_1, c_2 - constant of integration;

r_1, r_2 - characteristic coefficient of the equation:

$$r_{1,2} = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

Integration constants define using the initial conditions: $\tau = 0; t_3 = t_{30}; dt_3/d\tau = 0$. Substituting the values of the initial conditions in equation (37) and its derivative we will have:

$$\begin{cases} t_{30} = c_1 + c_2 + \frac{D_1(x)}{C} \\ 0 = c_1 r_1 + c_2 r_2 \end{cases} \tag{39}$$

Hence we have:

$$c_1 = -r_2 \frac{t_{30} C - D_1(x)}{C(r_1 - r_2)}; \quad c_2 = r_1 \frac{t_{30} C - D_1(x)}{C(r_1 - r_2)}$$

Thus the equation for the change in temperature of moving grain layer in time and coordinate will have the final form:

$$t_3(\tau, x) = r_1 \frac{t_{30} C - D_1(x)}{C(r_1 - r_2)} [r_1 e^{r_2 \tau} - r_2 e^{r_1 \tau}] + \frac{D_1(x)}{C} \tag{40}$$

Similarly we obtain the equation for the temperature change of rods $\Theta_c(\tau)$:

$$\Theta_c(\tau) = r_1 \frac{\Theta_{30} C - D_2}{C(r_1 - r_2)} [r_1 e^{r_2 \tau} - r_2 e^{r_1 \tau}] + \frac{D_2}{C}. \tag{41}$$

where:

$$D_2 = \frac{c_2}{a_2} + \frac{b_2}{a_2} c_3;$$

Θ_{30} - the initial temperature of the rods.

Using the resulting mathematical model (38) to identify the rape grain temperature in a channel with ferromagnetic rods [11, 17, 18], which are heated by the induction method [7, 10, 19]. Experimental data are given in [8, 9].

We substitute the thermal characteristics of rape grain from the source [20] to the obtained mathematical models (31) and (38). For the preliminary calculation we take the assumption that the material moves only under the influence of gravitational forces, and the ferromagnetic rods do not move. The result is the dependence of rape grain temperature change along the length of the unit frame (Fig. 1) and the dependence of rape grain temperature change of grain t_z and the heater temperature Θ_c in time (Fig. 2).

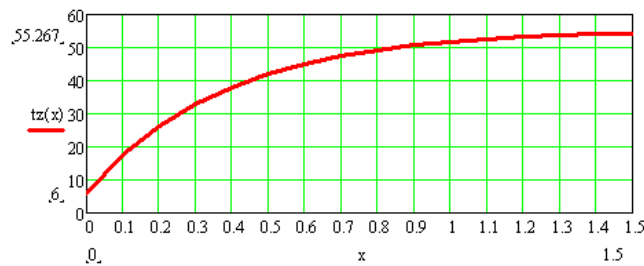


Fig. 1. The dependence of rape grain temperature change along the length of the unit (t_z , °C; x , m)

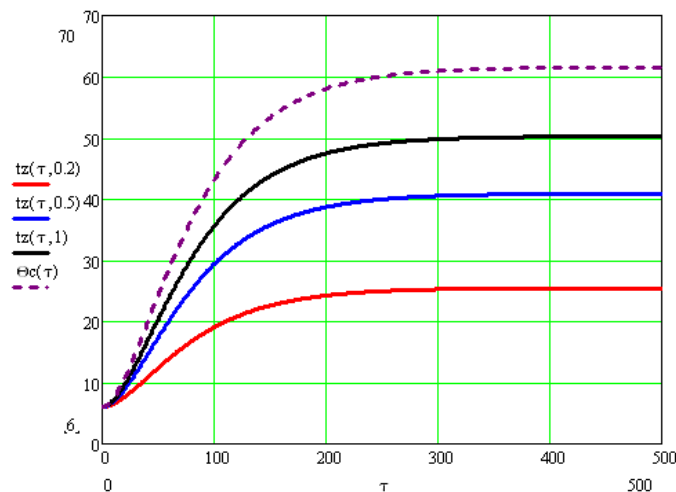


Fig. 2. Dependences of rape grain temperature change of grain t_z and the heater temperature Θ_c in time at distances 0.2, 0.5 and 1 m from the feeding tube of the unit (t_z , °C; Θ_c , °C; τ , s)

Conclusions

There are developed improved mathematical models of the dynamics of disperse material heating in a moving layer with uniformly placed heat-generating el-

ements with not previously taken into account factors (types of heat transfer, evaporation), which link the design and operational parameters of installation and allow us to determine the static and dynamic characteristics of the object.

The resulting mathematical model describes the process of rape grain heating with sufficient adequacy and allows you to determine the theoretical values of the grain temperature and the temperature of ferromagnetic rods for various technological modes of thermal processing installation operating.

References

- [1] Лисенко В.П., Запровадження енергоефективних комплексів в виробництві олії (стан питання), П.В. Лисенко, Д.С. Комарчук, Біоресурси і природокористування, НУБіП, К.: 2011, т. 3, № 1-2, с. 153-157.
- [2] Щербаков В.Г., Технология получения растительных масел, В.Г. Щербаков, Колос, М.: 1992, 326 с.
- [3] <http://lavrin-oil.ub.ua/ru/goods/view/21236/all/maslopres-shnekoviy-mmsh-220-z-pidigrivom-sirovini-skovoroda>
- [4] Бардин Я.Б., Ріпак: від сівби до переробки, Я.Б. Бардин, Світ, К.: 2000, 106 с.
- [5] Гайдаш В.Д., Ковальчук Г.М., Дем'янчук Г.Т. Ріпак - культура великих можливостей, Карпати, Ужгород 1986, 62 с.
- [6] Гайдаш В.Д., Ріпак, В.Д. Гайдаш, М.М. Климчук, М.М. Макара, Сіверсія, Івано-Франківськ 1998, 224 с.
- [7] Пат. 72273 UA, МПК F26B 11/00 (2011) Установка для термообработки олієнасіння, Комарчук Дмитро Сергійович, Лисенко Віталій Пилипович, Калініченко Роман Андрійович, Котов Борис Іванович; заявник і власник Національний університет біоресурсів і природокористування України, № u201101687; заявл. 15.02.2011; опубл. 10.08.2012, Бюл. № 15, 2012 р.
- [8] Кондратенко І.П., Дослідження розподілу температури в завантаженні циліндричного індуктора, І.П. Кондратенко, В.П. Лисенко, Д.С. Комарчук, Науковий вісник, НУБіП, К.: 2013, № 184, ч. 1, с. 74-82.
- [9] Кондратенко І.П., Індукційна установка для термообработки зерна ріпаку, І.П. Кондратенко, В.П. Лисенко, А.О. Березюк, Д.С. Комарчук, Вісник аграрної науки, Аграрна наука, К.: 2012, № 12, с. 55-58.
- [10] Пат. 66838 UA, МПК F26B 11/00 (2011) Установка для термообработки насіння, Комарчук Дмитро Сергійович, Лисенко Віталій Пилипович; заявник і власник Національний університет біоресурсів і природокористування України, № u201106029; заявл. 16.05.2011; опубл. 25.01.2012, Бюл. № 2, 2012 р.
- [11] Пат. 99666 UA, МПК F26B 11/00 (2011) Установка для термообработки олієнасіння, Комарчук Дмитро Сергійович, Лисенко Віталій Пилипович, Лук'янець Василь Олександрович; заявник і власник Національний університет біоресурсів і природокористування України, № a201101017; заявл. 31.01.2011; опубл. 10.09.2012, Бюл. № 17, 2012 р.
- [12] Crisp J., Woods J.L., The drying properties of rapeseed, J. Agric. Engng Res. 1994, 2, 89-97.
- [13] Касянчук В.Д., Семенова Л.Д., Переробка насіння на олію, Ріпак, Сіверсія ЛТД, Івано-Франківськ 1998, с. 189-205.
- [14] Масликов В.А., Технологическое оборудование производства растительных масел, В.А. Масликов, Пищевая пром-ть, М.: 1974, 500 с.
- [15] Лисенко В.П., Ідентифікація процесу нагріву зерна ріпаку, В.П. Лисенко, Р.А. Калініченко, Д.С. Комарчук, Науковий вісник, НУБіП, К.: 2012, № 174, ч. 1, с. 98-100.

- [16] Лисенко В.П., Математичне моделювання нестационарних теплових процесів прес-екструдера з індукційним обігрівом як об'єкта з розподіленими параметрами, В.П. Лисенко, Б.І. Котов, Д.С. Комарчук Р.А. Калініченко, Праці Таврійського державного агротехнічного університету, Мелітополь 2012, Том 2, № 12, с. 165-169.
- [17] Лисенко В.П., Дослідження технологічних параметрів екстругування олії при переробці ріпаку з використанням методів планування експерименту, В.П. Лисенко, В.О. Мірошник, Д.С. Комарчук, Науковий вісник, НУБіП, К.: 2011, № 166, ч. 3, с. 98-104.
- [18] Лисенко В.П., Математичне моделювання теплових процесів прес-екструдера з індукційним обігрівом, В.П. Лисенко, Б.І. Котов, Д.С. Комарчук, Науковий вісник, НУБіП, К.: 2011, № 166, ч. 4, с. 113-119.
- [19] Kawaguchi H., Enokizono M., Todaka T., Thermal and magnetic field analysis of induction heating problems, Journal of Materials Processing Technology 2005, 161, № 2, p. 193-198.
- [20] Гинзбург А.С., Теплофизические характеристики пищевых продуктов [Текст], А.С. Гинзбург, М.А. Громов, Г.И. Красовская, Пищевая пром-сть, М.: 1980, 288 с.
- [21] Котов Б.І., Ідентифікація динаміки електричних установок термообробки фуражного зена при виробництві, Б.І. Котов, В.П. Лисенко, Д.С. Комарчук Р.А., Калініченко, Науковий вісник Мелітополь, 2012, Том 4, № 2, с. 3-8.
- [22] Лисенко В.П., Інтенсифікація температурної обробки олійного насіння, В.П. Лисенко, Д.С. Комарчук, Науковий вісник, НУБіП, К.: 2012, № 161, с. 171-174.
- [23] Смирнов Н.В., Курс теории вероятностей и математической статистики для технических приложений, Н.В. Смирнов, И.В. Дунин-Барковский, Наука, М.: 1969, 512 с.

Abstract

The mathematical model of the rape grain heating process at conductive heat input using induction heating considering temperature distribution in the direction of the disperse material movement is developed.

Резюме

Создана математическая модель процесса нагрева зерна рапса при кондуктивном подводе теплоты с индукционным нагревом учитывая распределение температуры в направлении движения дисперсного материала.