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INTERPRETATION OF ISOTROPIC AND KINEMATIC HARDENING OF MATERIALS

The literature on different forms of hardening of materials is broad (see [1-25], for instance). The present paper analyses two basic types of hardening i.e. kinematic and isotropic. The interpretation of loading functions on plane is important for proper understanding of the problem, so in this paper this was the main assumption in preparing the manuscript. The stress $\boldsymbol{\sigma}$ at any loading state characterizes any open elementary system. The loading point ($\boldsymbol{\sigma}$) in the stress space {$\boldsymbol{\sigma}$} represents the present loading state. Denote the domain of elasticity in initial state by E_D. It contains the zero loading point ($\boldsymbol{\sigma}$) = (**0**). In the elasticity domain the strain increase remains reversible or elastic, for any path of the loading point ($\boldsymbol{\sigma}$) starting from the origin of space and lying inside this domain (Fig. 1).

A hardening frozen energy is absent in ideal plastic material without any hardening effect. The initial domain of elasticity for this material is not changed by the appearance of plastic strain. The elasticity domain is identical to the initial domain, and the loading point ($\boldsymbol{\sigma}$) cannot leave this domain (Fig. 1). If the loading point is and remains on the boundary of the elasticity domain E_D as illustrated by the loading path **12** in Figure 1. then the evolutions of plastic strain occur. Consider a loading path leaving the boundary towards the interior of domain E_D. It can be for instance the path **23** in Figure 1 corresponding to a purely elastic evolution of the elementary system. It corresponds to an elastic unloading.

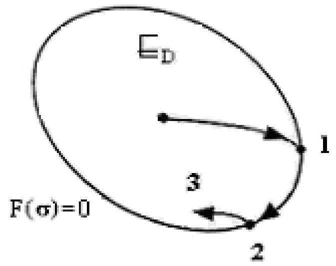

Fig. 1. Elasticity domain of ideal plastic material

The elasticity domain is defined by a scalar function F. It is called the loading function and has **σ** as its arguments. It is such that F (**σ**) < 0 represents the interior of domain E_D, F (**σ**) = 0 represents the boundary of domain E_D and F (**σ**) > 0 represents the exterior of domain E_D. The criterion F (**σ**) < 0 is the elasticity criterion. The criterion F (**σ**) = 0 is the plasticity criterion. The surface in the space of loading points {**σ**}, defined by F (**σ**) = 0, represents the boundary of domain E_D and is called the yield locus. The plastically admissible loading state (**σ**) satisfies the criterion F (**σ**) ≤ 0.

The elasticity domain for hardening materials is altered by the appearance of plastic strain. In the space {**σ**} of loading points, the present elasticity domain is defined as the domain arisen by the set of elastic unloading paths, or reversible loading paths, which issue from a present loading point **2**, as path **23** in Figure 2.

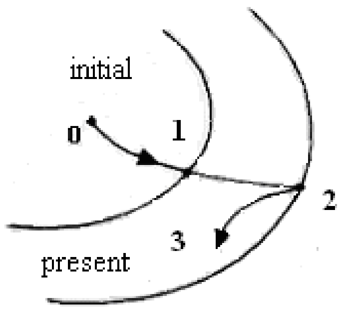

Fig. 2. Elasticity domains of hardening material

The present loading point is not necessarily on the boundary of the present elasticity domain, such as point **3** in Figure 2. There still exists an initial elasticity domain but, as soon as the loading point (**σ**) reaches for the first time the boundary of the initial elasticity domain (point **1**), further loading can deform this domain while carrying it along (loading path **12**). This is the phenomenon of hardening. The present elasticity domain depends not only on the present loading point (**σ**), but also on the loading path followed before, and thus on the hardening state.

Consider the domain of elasticity in the present state E_D. It is defined by a scalar loading function F, with arguments **σ** and with some hardening parameters represented by hardening force **η**. For the hardening material, it is such that F (**σ**, **η**) < 0 represents the interior of domain E_D, F (**σ**, **η**) = 0 represents the boundary of domain E_D, F (**σ**, **η**) > 0 represents to the exterior of domain E_D. The criterion of elasticity is expressed by F (**σ**, **η**) < 0. The plasticity threshold or criterion is expressed by F (**σ**, **η**) = 0. The surface defined by F (**σ**, **η**) = 0, in the space of loading points {**σ**}, representing the boundary of the present domain E_D is called the present yield locus.

We say that a loading state (**σ**) the plastically admissible in the present state if it satisfies the criterion F (**σ**, **η**) ≤ 0.

The loading function F considered in the space $\{\sigma \times \eta\}$ by the equation $F(\sigma, \eta) \leq 0$ defines a generalized elasticity domain E, which is now fixed as in ideal plasticity and which the generalized loading point (σ, η) cannot escape. The present elasticity domain E_D, as previously defined in the space $\{\sigma\}$, appears in the extended space $\{\sigma \times \eta\}$ as the intersection of the fixed domain E with the hyperplane $\eta = \eta_{present}$, where $\eta_{present}$ is the present value of hardening force η. Note that, owing to hardening phenomena, the origin O = (**0**) of space $\{\sigma\}$ may become outside the present elasticity domain, as illustrated in Figure 4.

Consider hardening parameters. A zero hardening force ($\eta = 0$) corresponds to a material state without any hardening history. The origin O = (**0**, **0**) in space $\{\sigma \times \eta\}$ necessarily belongs to the extended elasticity domain E, since it corresponds to a material without any loading history. The loading paths **01**, **12** and **23** in space $\{\sigma\}$, as represented in Figure 3 can be simply interpreted in the space $\{\sigma \times \eta\}$. The loading path **01** corresponds to an elastic evolution from the virgin state ($\sigma = \eta = 0$), without evolution of the hardening state ($d\eta = 0$). Therefore, the loading path **01** in the space $\{\sigma\}$ corresponds to the loading path **01** in the space $\{\sigma \times \eta\}$. At point **1** the plasticity criterion is satisfied, and loading path **12** corresponds to a plastic evolution during which the loading point carries along the elasticity domain E_D, while deforming it.

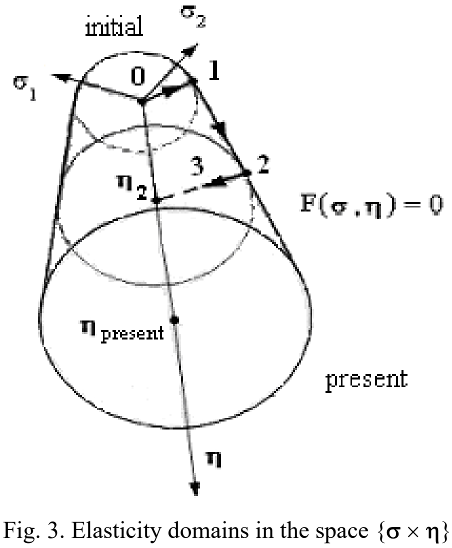

Fig. 3. Elasticity domains in the space $\{\sigma \times \eta\}$

The hardening state is modified and the plasticity criterion F = 0 is constantly satisfied during this evolution. Hence, in the space $\{\sigma \times \eta\}$ the loading point moves on the boundary of the fixed domain E from point **1** to point **2**. The loading path **23** corresponds to an elastic unloading without evolution of the hardening state ($d\eta = 0$). Therefore, the hardening force η keeps the value η_2 reached at point **2**. The loading path **23** in the space {σ} corresponds to the loading path **23** in the space {σ × η}, as illustrated in Figure 3.

The present elasticity domain E_D depends on the present value of the hardening force η. This dependency is the basis of their experimental identification. To be useful in practice, models must involve a few hardening variables, which correspond to a few components for vector η. For this purpose, simple hardening models have been designed.

The first one is the isotropic hardening model. In this model the elasticity domain in space {σ} is transformed by a homothety centred at the origin, as illustrated in Figure 4. The hardening force is reduced to a single scalar parameter η required to characterize this homothety.

The second one is the kinematic hardening model. In this model the boundaries in space {σ} of the elasticity domain are obtained through a translation of the boundary of the initial domain. The hardening variables are the variables characterizing this translation. They reduce to a tensor parameter η relative to the translation with respect to the stress tensor (Fig. 4). The two previous hardening models can also be combined to yield an isotropic and kinematic hardening model, as illustrated in Figure 4. As defined in this section, the hardening force η represents only a set of variables well suited for mathematical description of the observed evolution of the elasticity domain, and thus may not yet be considered as a thermodynamic force.

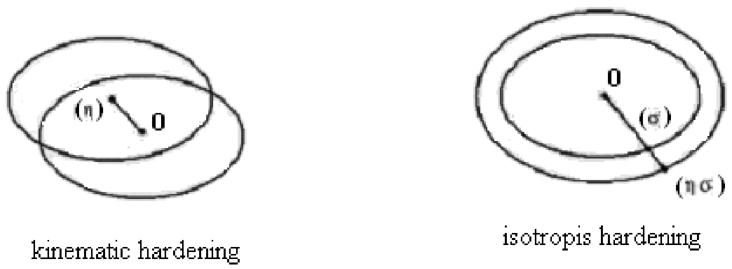

Fig. 4. Hardening models

The initial and present elasticity domains are convex. This property of convexity constitutes one of the sufficient criterions for the stability of plastic materials. In the loading point space {σ × η}, the fundamental geometrical property of a convex domain is that all points of a segment of a line that joins two points on the boundary of the domain lie inside this domain.

References

[1] Baltow A., Sawczuk A., A rule of anisotropic hardening, Acta Mechanika 1965, 2, 1-11.

[2] Bednarski T., Mechanics of plastic flow, Polish Scientific Publishers, Warsaw 1995.

[3] Drucker D.C., Some implications of work hardening and ideal plasticity, Quart Appl. Math. 1950, 7, 411-418.

[4] Drucker D.C., On uniqueness in the theory of plasticity, Quart Appl. Math. 1956, 14, 35-42.

[5] Drucker D.C., A definition of stable inelastic material, J. Appl. Mech. 1959, 26, 101-106.

[6] Drucker D.C., Prager W., Soil mechanics and plastic analysis of limit design, Quart Appl. Math. 1962, 10, 157-165.

[7] Drucker D.C., On the postulate of stability of material in the mechanics of continua, Journal de Mécanique 1964, 3, 235-249.

[8] Hill R., The mathematical theory of plasticity, Clarendon Press, Oxford 1956.

[9] Hoffman O., Sachs G., Introduction to the theory of plasticity for engineers, McGraw-Hill Book Company, New York 1953.

[10] Johnson W., Mellor P.B., Engineering plasticity, Van Nostrand Reinhold Company, London 1973.

[11] Malinin N.N., The theory of plasticity and creep, Maszinostrojenije, Moscow 1975.

[12] Marciniak Z., Mechanics of sheet forming, Polish Scientific Publishers, Warsaw 1961.

[13] Mises R., Mechanik der festen Körper in plastisch deformablem Zustand, Götinger Nachrichten, 1913.

[14] Perzyna P., Theory of viscoplasticity, Polish Scientific Publishers, Warsaw 1966.

[15] Perzyna P., Thermodynamics of non-elastic materials, Polish Scientific Publishers, Warsaw 1978.

[16] Pokorska I., Deformation of powder metallurgy materials in cold and hot forming, Journal of Materials Processing Technology 2008, 196, 1-3, 15-32.

[17] Pokorska I., Modeling of powder metallurgy processes, Advanced Powder Technology 2007, 18, 5, 503-539.

[18] Pokorska I., Modele sztywno-plastycznych porowatych materiałów budowlanych, ZN Politechniki Częstochowskiej 2004 nr 158, Budownictwo 10, 137-141.

[19] Pokorska I.. Modelling of isotropic and kinematic hardening in porous materials, ZN Politechniki Częstochowskiej 2004 nr 158, Budownictwo 10, 143-156.

[20] Pokorska I., Material hardening in porous lightweight structure undergoing thermomechanical loading, Lightweight Structures in Civil Engineering, International Seminar of IASS Polish Chapter, Warsaw 2004, 175-179.

[21] Sawczuk A., Mechanics and plasticity of structures, Polish Scientific Publishers and Ellis Horwood Limited 1989.

[22] Skrzypek J., Plasticity and Creep, Polish Scientific Publishers, Warsaw 1986.

[23] Slater R.A.C., Engineering plasticity, Wiley, New York-Toronto 1977.

[24] Sluzalec A., Theory of metal forming plasticity, Springer, 2004.

[25] Tayler G.J., Quinney H., The plastic distortion of metals, Phil. Trans. Roy. Soc. 1931, A230.

Abstract

The purpose of the paper is to discuss the hardening problems in materials. Kinematic and isotropic hardening are considered. The paper has a review character.

Streszczenie

W pracy przedyskutowano problemy wzmocnienia materiałów. Analizowano zarówno wzmocnienie kinematyczne, jak i izotropowe. Praca ma charakter przeglądowy.