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PRACTICAL METHOD FOR ANALYSIS OF ORTHOGONAL PLATES WITH CONSIDERATION OF SPECIFIC MODEL OF DESIGN DIAGRAM, BOUNDARY CONDITIONS AND LOADING

Introduction

Currently existing lot of methods for analysis of plates that are stipulated by versatility of boundary conditions and acting loadings, are characterized by labor consuming calculation process. Thus important is to develop such calculation methods that will be aimed for specific task (specific boundary conditions and specific type of loading) and gives the standard dependencies for significant simplifying of solution of task.

Basic part

Let's consider the orthogonal plate, edges of that are attached by arbitrary combination of hinged and rigid supports and on them is acting arbitrary loading.

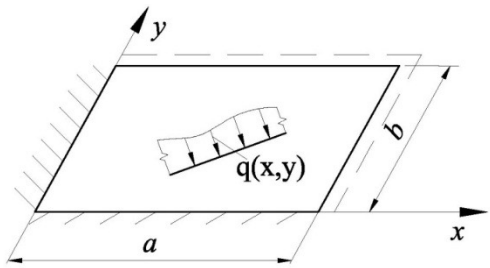

Fig. 1. Orthogonal plate those sides are attached by combination of hinged and rigid supports

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The deflection of such plate

$$W = W(x,y) = W_{max} f(x) \varphi(y) \quad (1)$$

will be satisfied the according conditions of attachment and in addition one more condition: when $x = \xi$ and $y = \eta$

$$W = W(x,y) = W_{max} \quad (2)$$

where ξ and η are corresponding normal coordinates of maximal deflection of plate.

For solution of task under study let's act as follow.

Lets divide the design diagram (Fig. 1) accordingly of boundary conditions on two sub-schemes (Fig. 2) that represents the beams. Due the conjugation of these beams deformations will be reached equivalence of α and β sub-schemes with given scheme. By conditions of this conjugation will be determined the force factor q* of interaction, by application of that on β sub-system (conditional beam) is possible to determine $W(x,y)$ because in appropriate cross-sections of conditional beams maximal deflections angles of rotation will be equal to zero (as well as in corresponding to maximal deflections of plate cross-sections), the condition of deformations conjugation for these cross-sections will be written down only by real deflections

$$\delta^{a}_{max} q^{*} - \delta^{a}_{max} q = \delta^{\beta}_{max} q^{*} \quad (3)$$

where δ^{a}_{max} and δ^{β}_{max} represents the maximal deflection caused due unit force factors in α and β sub-schemes of according beams.

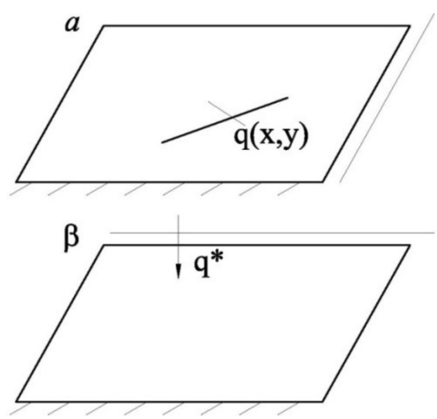

Fig. 2. Design diagram

By solution of equation (3) we will obtain

$$q^* = \frac{q}{1 + \frac{\delta_{max}^{\beta}}{\delta_{max}^{\alpha}}} \tag{4}$$

Thus, $W_{max} = \delta_{max}^{\beta} \cdot q^*$. As for functions $f(x)$ and $\varphi(x)$ they represents deflections of corresponding beams of α and β sub-systems (without physical parameters). By introduction of these functions in equation (1) and satisfaction of condition (2) will be obtain

$$W = W(x,y) = W_{max}\frac{f(x)\varphi(y)}{f(\xi)\varphi(\eta)} \tag{5}$$

The further calculations (determination of bending and torque moments, shear forces and so on) will be carried out by known formulae of classical theory of elasticity [1].

For illustration of offered method, let's consider the specific cases.

Case 1. In the center of hinged attached by all four sides plate is applied concentrated force F = const (Fig. 3).

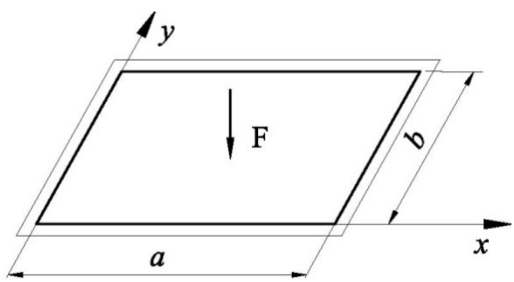

Fig. 3. The hinged attached by all four sides plate

According of [2] for hinged supported on two supports beam, in mid-span of that is acting concentrated force, the deflection function will be as:

$$\Delta^{\alpha}(x) = \frac{F}{12EI^{\alpha}}\left(x^3 + \frac{a^2}{4}x\right); \quad \left[0 \le x \le \frac{a}{2}\right] \tag{6}$$

The maximal deflection in the mid-span will be

$$\Delta_{max}^{\alpha} = \frac{Fa^3}{48EI^{\alpha}} = 0.028\frac{Fa^3}{EI^{\alpha}} \tag{7}$$

By consideration of (7) we will obtain:

$$\delta_{max}^{\alpha} = 0.0208\frac{a^3}{EI^{\alpha}}; \quad \delta_{max}^{\beta} = 0.0208\frac{b^3}{EI^{\beta}} \tag{8}$$

It is known that cylindrical stiffness of plate

$$D = E\frac{h^3}{12(1-v^2)} \tag{9}$$

where h is the thickness of plate and

$$EI^{\alpha} = E\frac{bh^3}{12} \quad \text{and} \quad EI^{\beta} = E\frac{ah^3}{12} \tag{10}$$

Therefore:

$$\frac{EI^{\alpha}}{D}(1-v^2)b \quad \text{and} \quad \frac{EI^{\beta}}{D}(1-v^2)a \tag{11}$$

Whence will be obtained

$$EI^{\alpha} = (1-v^2)bD \quad and \quad EI^{\beta} = (1-v^2)aD \tag{12}$$

By introduction of (8) in (11) we will obtain:

$$\begin{gathered} \delta^{\alpha}_{max} = 0.0208\frac{a^3}{(1-v^2)bD} \\ \delta^{\beta}_{max} = 0.0208\frac{b^3}{(1-v^2)aD} \end{gathered} \tag{13}$$

Due taking into account (13) the expression (4) will be as

$$F^* = \frac{F}{1+\dfrac{b^4}{a^4}} \tag{14}$$

Thus the maximal deflection in corresponding beam of β sub-system that is equivalent of given plate (Fig. 3) maximal deflection, will be as:

$$\Delta^{\alpha}_{max} = W_{max} = \frac{0.0208Fb^3}{\left(1+\dfrac{b^4}{a^4}\right)(1-v^2)aD} \tag{15}$$

Accordingly of (6)

$$f(x) = x^3 + \frac{a^2}{4}x; \quad \varphi(y) = y^3 + \frac{b^2}{4}x \tag{16}$$

The maximal deflection is in the center of plate, thus $\xi = \frac{a}{4}$ and $\eta = \frac{b}{4}$, i.e.

$$f(\xi) = \frac{a^3}{4} \quad \text{and} \quad \varphi(\eta) = \frac{b^3}{4} \tag{17}$$

By introduction of (15) and (16) in the expression (17):

$$W = W(x,y) = \frac{0.0208Fb^3}{\left(1+\frac{b^4}{a^4}\right)(1-v^2)aD}\frac{16}{a^3b^3}\left(x^3+\frac{a^2}{4}x\right)\left(y^3+\frac{b^2}{4}y\right) \quad (18)$$

At $a = b$ and $v = 0.3$ from (16) we will obtain:

$$W_{max} = \frac{0.0208F}{2}\frac{a^2}{0.91D} = 0.0114\frac{Fa^2}{D}$$

Accordingly of [3]

$$W_{max} = 0.0116\frac{Fa^2}{D}$$

As we see the results practically coincide to each other.

Case 2. All four sides of plate are hinged attached and on them is applied uniformly distributed loading q = const.

In this case accordingly of [4] we will have

$$\Delta^{\alpha}(x) = \frac{qb}{24EI^{\alpha}}(x^4 - 2ax^3 + a^2x^2) \quad (19)$$

The maximal deflection in the mid-span will be

$$\Delta^{\alpha}_{max} = \frac{dba^4}{384EI^{\alpha}} = 0.0026\frac{qba^4}{EI^{\alpha}} \quad (20)$$

By consideration of (19) we will obtain:

$$\delta^{\alpha}_{max} = 0.0026\frac{a^4}{(1-v^2)bD}$$
$$\delta^{\beta}_{max} = 0.0026\frac{b^4}{(1-v^2)aD} \quad (21)$$

Due taking into account (21) will be obtained

$$q^* = \frac{q}{1+\frac{b^4}{a^4}} \quad (22)$$

Thus the maximal deflection in corresponding beam of β sub-system that is equivalent of given plate (Fig. 4) maximal deflection, will be as:

$$\Delta^{\alpha}_{max} = W_{max} = \frac{0.0026qb^4}{\left(1+\frac{b^4}{a^4}\right)(1-v^2)\cdot D} \quad (23)$$

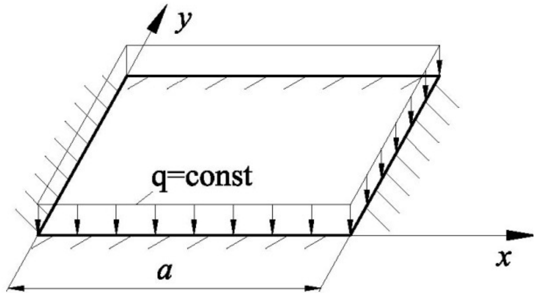

Fig. 4. Plate with maximal deflection

Accordingly of (18)

$$f(x) = x^4 - 2ax^3 + a^2x^2$$
$$\varphi(y) = y^4 - 2bx^3 + b^2y^2 \quad (24)$$

The maximal deflection is in the center of plate, thus $\xi = \frac{a}{2}$ and $\eta = \frac{b}{2}$, i.e.

$$f(\xi) = \frac{a^4}{16} \text{ and } \varphi(\eta) = \frac{b^4}{16} \quad (25)$$

By introduction of (23), (24) and (25) in the expression (1):

$$W = W(x,y) = \frac{0.0026Fb^4}{\left(1 + \frac{b^4}{a^4}\right)(1 - v^2)aD} \frac{256}{a^4b^4} (x^4 - 2ax^3 + a^2x^2)(y^4 - 2bx^3 + b^2y^2) \quad (26)$$

At $a = b$ and $v = 0.3$ from (24) we will obtain:

$$W_{max} = \frac{0.0026qa^2}{2 \cdot 0.91D} = 0.0014 \frac{qa^4}{D}$$

For calculated by Bubnov-Galerkin method such plate

$$W_{max} = 0.0013 \frac{qa^4}{D}$$

i.e. in this case the results are practically coincides to each other.

Conclusion

The paper present the practical method for analysis on bending of orthogonal plates with consideration of various boundary conditions and versatility of loadings. There are carried out calculations for all possible cases of side's attachment.

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Abstract

Is stated practical method for analysis of orthogonal plates with consideration of specific model of design diagram, boundary conditions and loading. For illustration of offered method are considered specific examples: applied in the center pinning on all four sides plate, when is constant, all four sides of plate are rigidly attached and on them is applied uniformly distributed constant loading.

Keywords: plate, hinge, simultaneous equations; design diagram; boundary conditions

Praktyczna metoda analizy ortogonalnych płyt z uwzględnieniem schematu statycznego, warunków brzegowych i obciążenia

Streszczenie

W artykule przedstawiono metodę analizy ortogonalnych płyt, która ma zastosowanie praktyczne. Metoda projektowania polega na analizie schematu statycznego, warunków brzegowych i obciążeń. W celu zilustrowania metody przedstawiono przykłady obliczeniowe. Pierwszy dotyczy płyty opartej swobodnie na wszystkich czterech krawędziach, obciążonej siłą skupioną pośrodku, natomiast drugi dotyczy płyty zamocowanej sztywno oraz obciążonej obciążeniem równomiernie rozłożonym.

Słowa kluczowe: płyta, przegub, układ równań, schemat statyczny, warunki brzegowe